Supplementary Material for "Hybrid and Oriented Harmonic Potentials for Safe Task Execution in Unknown Environment"

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Abstract

This document contains the additional definitions, proofs, algorithmic and experiment details that are omitted in the main manuscript of "Hybrid and Oriented Harmonic Potentials for Safe Task Execution in Unknown Environment", due to limited space. References to the main manuscript including equations, definitions, lemmas, theorems are marked by "P[X]".

1. Definition of Contraction-like Transformation

The transformation $\Phi_{\mathcal{M}\to\tilde{\mathcal{P}}}(q)$ form a sphere world $\mathcal{M}$ to a bounded point world in P[1] is given by $\tilde{\mathcal{P}} = O_0\backslash\{\tilde{P}_1, \cdots, \tilde{P}_M\}$.

$$\begin{aligned}
&\Phi_{\mathcal{M}\to\mathcal{P}}(q) \triangleq \psi \circ \Phi_{\mathcal{M}\to\tilde{\mathcal{P}}}(q),\\
&\Phi_{\mathcal{M}\to\tilde{\mathcal{P}}}(q) \triangleq \mathrm{id}(q) + \sum_{i=1}^{M}\big(1 - s_\delta(q,\, O_i)\big)(q_i - q),\\
&\psi(\tilde{q}) \triangleq \frac{\rho_0}{\rho_0 - ||\tilde{q} - q_0||}(\tilde{q} - q_0) + q_0,
\end{aligned} \tag{1}$$

where $s_\delta(q,\, O_i)$ is the contraction-like transformation for obstacle O_i, which is composed by $\eta_\delta(x) \circ \sigma(x) \circ b_i(x)$ as the switch function, smoothing function and distance function, respectively. The specifical definitions is introduced in Loizou & Rimon (2022,2021) and defined as follows.

$$\eta_\delta(x) \triangleq \frac{\sigma(x)}{\sigma(x) + \sigma(\delta - x)};$$

$$\sigma(x) \triangleq \begin{cases} e^{-1/x}, & x > 0 \\ 0, & x \leqslant 0 \end{cases};$$

and

$$b_i(x) \triangleq \|x - P_i\| - \rho_i,$$

where $P_i \in \mathbb{R}^2$ is the position of the point obstacle $\mathcal{O}_i$ and ρ_i is the radius of the sphere obstacle O_i.

2. Adaptive Obstacle Estimation

The adaptive obstacle estimation consists of three steps: clustering, fitting, and decomposition. This sections will provide technical details and practical results related to these steps.

Clustering: Given a 2D point cloud $\mathbf{Q}_t = \{q_1, q_2, \cdots, q_N\}$ obtained from the LiDAR sensor at the current time step t, the data points are divided into K clusters:

$$\mathbf{D}_t = \{\mathbf{D}_{t,1}, \mathbf{D}_{t,2}, \cdots, \mathbf{D}_{t,K}\}.$$

This partitioning is achieved by examining the relative distances between consecutive points, such that:

$$d(q_i, q_j) < \bar{\delta}_1,\ \forall\, \mathbf{D}_{t,k} \in \mathbf{D}_t,\ \forall\, q_i, q_j \in \mathbf{D}_{t,k}$$

and

$$\min_{p_\ell \in \mathbf{D}_{t,i};\, p_\iota \in \mathbf{D}_{t,j}} d(p_\ell, p_\iota) > \bar{\delta}_2,\ \forall\, \mathbf{D}_{t,i}, \mathbf{D}_{t,j} \in \mathbf{D}_t,$$

where $d(q_i, q_j)$ is the distance between q_i and q_j; $\bar{\delta}_1$ and $\bar{\delta}_2$ are thresholds. This clustering process can be effectively addressed using algorithms such as k-means or DBSCAN.

Fitting: Each cluster of point clouds can be fitted to a squircle using the least squares method, formulated as a constrained optimization problem:

$$\begin{aligned}
\mathop{\mathbf{min}}_{\Theta_k, \kappa_k} \quad & \sum_{q_\ell \in \mathbf{D}_{t,k}} \big(\beta_{\mathrm{sc}}(q_\ell, \Theta_k, \kappa_k)\big)^2 \\
\mathbf{s.t.} \quad & \bar{\Theta}_1 \preceq \Theta_k \preceq \bar{\Theta}_2, \\
& 0 < \kappa_k < 1,
\end{aligned}$$

where $\Theta_k \triangleq [c_k, w_k, h_k, \theta_k]$ represents the geometric parameters; c_k, w_k, h_k, θ_k and κ_k denote the center, width, height, orientation, and curvature of the squircle, respectively, $\forall k = 1, 2, \cdots, K$; $\bar{\Theta}_1$ and $\bar{\Theta}_2$ are the lower and upper bounds, respectively. This optimization problem can be solved via general nonlinear optimization solvers. Denote the optimal solution as $\Theta_k^\star$ and $\kappa_k^\star$; then, the fitting error is given by:

$$E_k = \sum_{q_\ell \in \mathbf{D}_{t,k}} \big(\beta_{\mathrm{sc}}(q_\ell, \Theta_k^\star, \kappa_k^\star)\big)^2.$$

If the fitting error E_k is smaller than the threshold e_k, the estimated model is accepted. Since the optimization problem is

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nonlinear, there may be multiple estimated models with acceptable fitting errors; the one with the smallest area is selected.

Decomposition: If the fitting error E_k exceeds the error threshold e_k, the cluster $\mathbf{D}_{t,k}$ is decomposed into multiple segments based on the curvatures, $\forall k = 1, 2, \cdots, K$. Specifically, denote the set of points near $q_i \in \mathbf{D}_{t,k}$ as $\mathcal{N}_i^k = \{q_j \in \mathbf{D}_{t,k} \,|\, d(q_i, q_j) < \bar{r}\}$, where $\bar{r}$ is the radius. A quadratic function $w_i^k(x)$ is then employed to fit the set of points $\mathcal{N}_i^k$. The curvature for each point $q_i \in \mathbf{D}_{t,k}$ is computed as follows:

$$\tilde{\kappa}_i^k = \frac{w_i^k(x)''}{[1 + (w_i^k(x)')^2]^{3/2}}.$$

Next, the points in the cluster $\mathbf{D}_{t,k}$ are further decomposed into serval segments $\mathbf{D}_{t,k} = \{\mathbf{S}_{t,1}, \mathbf{S}_{t,2}, \cdots, \mathbf{S}_{t,L}\}$ based on the curvatures $\tilde{\kappa}_i^k$, $\forall q_i \in \mathbf{D}_{t,k}$. Similarly, if the estimated squircle overlaps with the robot or the regions of interest, the cluster should also be further decomposed. Then, each of the segments $\mathbf{S}_{t,\ell}$ is then fitted to a squircle model, $\forall \ell = 1, 2, \cdots, L$. This process is repeated until all segments are fitted to squircles with acceptable errors.

3. Online Computation of Harmonic Potentials

3.1. Independent Stars

Lemma 5. *Each time an independent obstacle $\mathcal{O}_{k+1}$ is added to the workspace, the following holds:*

(i) the online omitted product for $\mathcal{O}_0$ is given by

$$\overline{\beta}_0^{k+1}(q) = \overline{\beta}_0^k(q)\, \beta_{k+1}(q);$$

(ii) the online analytic switch for $\mathcal{O}_0$ is given by

$$s_0^{k+1}(q) = \frac{s_0^k(q)}{\alpha^k(q)\,(1 - s_0^k(q)) + s_0^k(q)},$$

where $\alpha^k(q) = \lambda_{k+1}/\big(\lambda_k\, \beta_{k+1}(q)\big)$.

Proof. (i) By Definition P[4], W

$$\overline{\beta}_0^k(q) = \prod_{j=0, j\neq i}^{k} \beta_j(q)$$

at step-k and

$$\overline{\beta}_0^{k+1}(q) = \prod_{j=0, j\neq i}^{k+1} \beta_j(q)$$

at step-$(k+1)$. Comparing these two equations, it follows that

$$\overline{\beta}_0^{k+1}(q) = \overline{\beta}_0^k(q)\, \beta_{k+1}(q).$$

(ii) By Definition P[5],

$$s_0^k(q) = \frac{\gamma_G(q)\, \overline{\beta}_0^k(q)}{\lambda_k\, \beta_0(q) + \gamma_G(q)\, \overline{\beta}_0^k(q)},$$

which can be rearranged as

$$\overline{\beta}_0^k(q) = \frac{\lambda_k\, s_0^k(q)\, \beta_0(q)}{(1 - s_0^k(q))\, \gamma_G(q)}.$$

Further by Definition P[5] and (i),

$$\begin{aligned} s_0^{k+1}(q) &= \frac{\gamma_G(q)\, \overline{\beta}_0^{k+1}(q)}{\lambda_{k+1}\, \beta_0(q) + \gamma_G(q)\, \overline{\beta}_0^{k+1}(q)} \\ &= \frac{\gamma_G(q)\, \beta_{k+1}(q)\, \overline{\beta}_0^k(q)}{\lambda_{k+1}\, \beta_0(q) + \gamma_G(q)\, \beta_{k+1}(q)\, \overline{\beta}_0^k(q)}. \end{aligned}$$

Substituting the expression for $\overline{\beta}_0^k(q)$ derived earlier, it holds that

$$\begin{aligned} s_0^{k+1}(q) &= \frac{\gamma_G(q)\, \beta_{k+1}(q)\, \frac{\lambda_k\, s_0^k(q)\, \beta_0(q)}{(1 - s_0^k(q))\, \gamma_G(q)}}{\lambda_{k+1}\, \beta_0(q) + \gamma_G(q)\, \beta_{k+1}(q)\, \frac{\lambda_k\, s_0^k(q)\, \beta_0(q)}{(1 - s_0^k(q))\, \gamma_G(q)}} \\ &= \frac{\lambda_k\, s_0^k(q)\, \beta_{k+1}(q)}{\lambda_{k+1}\, [1 - s_0^k(q)] + \lambda_k\, s_0^k(q)\, \beta_{k+1}(q)}. \end{aligned}$$

Finally, considering

$$\alpha^k(q) = \lambda_{k+1}/[\lambda_k\, \beta_{k+1}(q)],$$

it follows

$$s_0^{k+1}(q) = \frac{s_0^k(q)}{\alpha^k(q)\, [1 - s_0^k(q)] + s_0^k(q)}.$$

□

Lemma 6. *Each time an independent obstacle $\mathcal{O}_{k+1}$ is added to the workspace, the following holds:*

(i) the online omitted product for $\mathcal{O}_{i+1}$ is given by

$$\overline{\beta}_{i+1}^{k+1}(q) = \overline{\beta}_i^{k+1}(q)\, \frac{\beta_i(q)}{\beta_{i+1}(q)};$$

(ii) the online analytic switch for $\mathcal{O}_{i+1}$ is given by

$$s_{i+1}^{k+1}(q) = \frac{s_i^{k+1}(q)}{\alpha_i(q)\,(1 - s_i^{k+1}(q)) + s_i^{k+1}(q)},$$

where

$$\alpha_i(q) = (\beta_{i+1}(q))^2/(\beta_i(q))^2.$$

Proof. (i) By Definition P[4],

$$\overline{\beta}_i^{k+1}(q) = \prod_{j=0, j\neq i}^{k+1} \beta_j(q)$$

and

$$\overline{\beta}_{i+1}^{k+1}(q) = \prod_{j=0, j\neq i+1}^{k+1} \beta_j(q).$$

Comparing these two equations, it follows that

$$\overline{\beta}_{i+1}^{k+1}(q) = \overline{\beta}_i^{k+1}(q)\, \frac{\beta_i(q)}{\beta_{i+1}(q)}.$$

(ii) By Definition P[5],

$$s_i^{k+1}(q) = \frac{\gamma_G(q)\,\overline{\beta}_i^{k+1}(q)}{\lambda_{k+1}\,\beta_i(q) + \gamma_G(q)\,\overline{\beta}_i^{k+1}(q)},$$

which can be rearranged as

$$\overline{\beta}_i^{k+1}(q) = \frac{\lambda_{k+1}\,s_i^{k+1}(q)\,\beta_i(q)}{(1 - s_i^{k+1}(q))\,\gamma_G(q)}.$$

Further by Definition P[5] and (i),

$$\begin{aligned} s_{i+1}^{k+1}(q) &= \frac{\gamma_G(q)\,\overline{\beta}_{i+1}^{k+1}(q)}{\lambda_{k+1}\,\beta_{i+1}(q) + \gamma_G(q)\,\overline{\beta}_{i+1}^{k+1}(q)} \\ &= \frac{\gamma_G(q)\,(\beta_i(q))^2\,\overline{\beta}_i^{k+1}(q)}{\lambda_{k+1}\,[\beta_{i+1}(q)]^2 + \gamma_G(q)\,(\beta_i(q))^2\,\overline{\beta}_i^{k+1}(q)}. \end{aligned}$$

Substituting the expression for $\overline{\beta}_0^k(q)$ derived earlier, it holds that

$$\begin{aligned} s_{i+1}^{k+1}(q) &= \frac{\gamma_G(q)\,(\beta_i(q))^2\,\frac{\lambda_{k+1}\,s_i^{k+1}(q)\,\beta_i(q)}{(1-s_i^{k+1}(q))\,\gamma_G(q)}}{\lambda_{k+1}\,(\beta_{i+1}(q))^2 + \gamma_G(q)\,(\beta_i(q))^2\,\frac{\lambda_{k+1}\,s_i^{k+1}(q)\,\beta_i(q)}{(1-s_i^{k+1}(q))\,\gamma_G(q)}} \\ &= \frac{s_i^{k+1}(q)\,(\beta_i(q))^2}{(1 - s_i^{k+1}(q))\,(\beta_{i+1}(q))^2 + s_i^{k+1}(q)\,(\beta_i(q))^2}. \end{aligned}$$

Finally, considering

$$\alpha_i(q) = (\beta_{i+1}(q))^2/(\beta_i(q))^2,$$

it follows

$$s_{i+1}^{k+1}(q) = \frac{s_i^{k+1}(q)}{\alpha_i(q)\,(1 - s_i^{k+1}(q)) + s_i^{k+1}(q)}.$$

□

3.2. Overlapping Stars

Lemma 7. *Each time an obstacle $\mathcal{O}_{k+1}$ is added to the workspace and overlapping with an existing obstacle, the following holds:*

(i) the online omitted product for $\mathcal{O}_{k+1}$ is given by

$$\overline{\beta}_{k+1}(q) = \overline{\beta}_k(q)\,\frac{(\beta_k(q))^2\,\tilde{\beta}_{k+1}(q)\,\beta_{k^\star}(q)}{\tilde{\beta}_k(q)\,\beta_{(k+1)^\star}(q)};$$

(ii) the online analytic switch for $\mathcal{O}_{k+1}$ is given by:

$$\sigma_{k+1}(q) = \frac{\sigma_k(q)}{\alpha_k(q)(1 - \sigma_k(q)) + \sigma_k(q)},$$

where

$$\alpha_k(q) = \frac{\xi_{k+1}\,\beta_{k+1}(q)\,\tilde{\beta}_k(q)\,\beta_{(k+1)^\star}(q)}{\xi_k\,(\beta_k(q))^3\,\tilde{\beta}_{k+1}(q)\,\beta_{k^\star}(q)}.$$

Proof. (i) By Definition P[6],

$$\overline{\beta}_{k+1}(q) \triangleq \Big(\prod_{j=0, j\neq(k+1)^\star}^{k} \beta_j(q)\Big)\Big(\prod_{j\in\mathcal{L}\setminus\{k+1\}} \beta_j(q)\Big)\tilde{\beta}_{k+1}(q),$$

and

$$\overline{\beta}_k(q) \triangleq \Big(\prod_{j=0, j\neq k^\star}^{k-1} \beta_j(q)\Big)\Big(\prod_{j\in\mathcal{L}\setminus\{k\}} \beta_j(q)\Big)\tilde{\beta}_k(q).$$

Comparing these two equations, it follows that

$$\overline{\beta}_{k+1}(q) = \overline{\beta}_k(q)\,\frac{(\beta_k(q))^2\,\tilde{\beta}_{k+1}(q)\,\beta_{k^\star}(q)}{\tilde{\beta}_k(q)\,\beta_{(k+1)^\star}(q)}.$$

(ii) By Definition P[7],

$$\sigma_k(q) = \frac{\gamma_G(q)\,\overline{\beta}_k(q)}{\xi_k\,\beta_k(q) + \gamma_G(q)\,\overline{\beta}_k(q)},$$

which can be rearranged as

$$\overline{\beta}_k(q) = \frac{\xi_k\,\sigma_k(q)\,\beta_k(q)}{(1 - \sigma_k(q))\,\gamma_G(q)}.$$

Further by Definition P[7] and (i),

$$\begin{aligned} \sigma_{k+1}(q) &= \frac{\gamma_G(q)\,\overline{\beta}_{k+1}(q)}{\xi_{k+1}\,\beta_{k+1}(q) + \gamma_G(q)\,\overline{\beta}_{k+1}(q)} \\ &= \frac{\gamma_G(q)\,\xi_k\,[\beta_k(q)]^3\,\tilde{\beta}_{k+1}(q)\,\beta_{k^\star}(q)\,\overline{\beta}_k(q)}{\Big(\xi_{k+1}\,\beta_{k+1}(q)\,\tilde{\beta}_k(q)\,\beta_{(k+1)^\star}(q) + \gamma_G(q)\,\xi_k\,[\beta_k(q)]^3\,\tilde{\beta}_{k+1}(q)\,\beta_{k^\star}(q)\,\overline{\beta}_k(q)\Big)} \end{aligned}$$

Substituting $\overline{\beta}_k(q)$ results that

$$\sigma_{k+1}(q) = \frac{\sigma_k(q)}{\alpha_k(q)\,[1 - \sigma_k(q)] + \sigma_k(q)}$$

with

$$\alpha_k(q) = \frac{\xi_{k+1}\,\beta_{k+1}(q)\,\tilde{\beta}_k(q)\,\beta_{(k+1)^\star}(q)}{\xi_k\,[\beta_k(q)]^3\,\tilde{\beta}_{k+1}(q)\,\beta_{k^\star}(q)}.$$

□